SWR and BANDWIDTH OF SERIES / PARALLEL R L C

1- Series R L C circuit - Calculations and Simulations

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$$\boxed{Z = j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C} + R}$$ Impedance of a series R L C — Eq 1

$$Q = \frac{\omega\cdot L}{R} \qquad \text{Then : } \quad R = \frac{\omega\cdot L}{Q}$$ Q factors (unloaded !) — Eq 2

$$Q = \frac{1}{\omega o\cdot C\cdot R} \qquad \text{Then : } \quad R = \frac{1}{\omega o\cdot C\cdot Q}$$ ωo is the resonant frequency — Eq 3

The impedance in Eq 1 is normalized

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{L}{R} + \frac{1}{j\cdot\omega\cdot C\cdot R} + 1$$ Eq 4

$$R = \frac{\omega o\cdot L}{Q} = \frac{1}{\omega o\cdot C\cdot Q}$$ Eq 5

Substitute Eq 5 into Eq 4:

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{L\cdot Q}{\omega o\cdot L} + \frac{\omega o\cdot C\cdot Q}{j\cdot\omega\cdot C} + 1$$ Eq 6

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{Q}{\omega o} + \frac{\omega o\cdot Q}{j\cdot\omega} + 1$$ After simplification — Eq 7

$$\frac{Z}{R} = j\cdot Q\cdot\left(\frac{\omega}{\omega o} - \frac{\omega o}{\omega}\right) + 1$$ Note that at resonance $\omega = \omega o$ and $Z / R = 1$ — Eq 8

At ω_H and ω_L the half power frequencies, the magnitude of Z/R equals $1 + j$
Therefore

$$\left|Q\cdot\left(\frac{\omega_H}{\omega o} - \frac{\omega o}{\omega_H}\right)\right| = \left|Q\cdot\left(\frac{\omega_L}{\omega o} - \frac{\omega o}{\omega_L}\right)\right| = 1$$ Eq 9

Then

$$\frac{\omega_H}{\omega o} - \frac{\omega o}{\omega_H} = \frac{\omega o}{\omega_L} - \frac{\omega_L}{\omega o}$$ Eq 10

Rearranging the terms:

$$\frac{\omega_H}{\omega o} + \frac{\omega_L}{\omega o} = \frac{\omega o}{\omega_L} + \frac{\omega o}{\omega_H}$$ Eq 11

$$\frac{\omega_H + \omega_L}{\omega o} = \frac{\omega o \cdot (\omega_H + \omega_L)}{\omega_H \cdot \omega_L} \qquad \text{Eq 12}$$

Therefore

$$\omega_H \cdot \omega_L = \omega o^2 \qquad \text{Eq 13}$$

$$\omega o = \sqrt{\omega_H \cdot \omega_L} \qquad \text{Eq 14}$$

Rearranging the first term of eq 9:

$$Q \cdot \frac{(\omega_H)^2 - \omega o^2}{\omega o \cdot \omega_H} = 1 \qquad \text{Eq 15}$$

From a combinaison of Eq 13 and Eq 15 :

$$Q \cdot \frac{(\omega_H)^2 - \omega_H \cdot \omega_L}{\omega o \cdot \omega_H} = 1 \qquad \text{Eq 16}$$

$$Q \cdot \frac{\omega_H - \omega_L}{\omega o} = 1 \qquad \text{Eq 17}$$

Then

$$\omega_H - \omega_L = \frac{\omega o}{Q} \qquad \text{Eq 18}$$

Converting to frequency:

$$f_H - f_L = \frac{f_o}{Q} \qquad \text{Eq 19}$$

f_o is the resonant frequency

f_H and f_L are the half power frequencies

The bandwidth BW in Hz is defined as:

$$BW = f_H - f_L \qquad \text{Eq 20}$$

Then from Eq. 19

$$BW = \frac{f_o}{Q} \qquad \text{And:} \qquad Q = \frac{f_o}{BW} \qquad \text{Eq 21a, 21b}$$

Eq 8 repeated

$$\frac{Z}{R} = j \cdot Q \cdot \left(\frac{\omega}{\omega o} - \frac{\omega o}{\omega}\right) + 1 \qquad \text{Eq 8}$$

For the SERIES RLC circuit :

We have established that at $\omega = \omega_H$ and $\omega = \omega_L$ the **normalized** impedance is:

$$\frac{Z}{R} = 1 + j \qquad \text{or} \qquad \frac{Z}{R} = 1 - j$$

Note that at resonance Z / R = 1 from Eq. 8.
This is the minimum value.

The reflection coefficient at $\omega = \omega_H$ and $\omega = \omega_L$ is:

These are calculated with respect to the normalized Zo value of 1

$$\text{rho} := \frac{(1+j)-1}{(1+j)+1} \qquad \text{rho} = 0.2 + 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

$$\text{rho} := \frac{(1-j)-1}{(1-j)+1} \qquad \text{rho} = 0.2 - 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

For the PARALLEL RLC circuit :

At $\omega = \omega_H$ and $\omega = \omega_L$ the **normalized** impedance Z/R is **1** in parallel with **+ / - j**:

Combining these impedances in parallel:

$$\frac{Z}{R} = \frac{1 \cdot j}{1+j} = 0.5 + 0.5i$$

$$\frac{Z}{R} = \frac{1 \cdot (-j)}{1-j} = 0.5 - 0.5i$$

The reflection coefficient at $\omega = \omega_H$ and $\omega = \omega_L$ is:

$$\text{rho} := \frac{(0.5+0.5i)-1}{(0.5+0.5i)+1} \qquad \text{rho} = -0.2 + 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

$$\text{rho} := \frac{(0.5-0.5i)-1}{(0.5-0.5i)+1} \qquad \text{rho} = -0.2 - 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

For BOTH series and parallel circuits, the reflection coefficients, return loss and SWR all have the same absolute value at the 3 dB points, where $\omega = \omega_H$ and $\omega = \omega_L$

$$\text{RL} := -20 \cdot \log(|\text{rho}|) \qquad \text{RL} = 6.99$$ **Return Loss**

$$\text{SWRt} := \frac{1 + |\text{rho}|}{1 - |\text{rho}|} \qquad \text{SWRt} = 2.61803$$ **SWR Reference Value**

Example of R L C SERIES CIRCUIT

$fr := 10$ MHz $\quad L := 100$ uH $\quad R := 50$ ohms $\quad Zo := R$

$$C := \frac{10^6}{(2\cdot\pi\cdot fr)^2\cdot L}$$

$C = 2.533$ $\quad C$ in pF is calculated to resonate at fr

$XL(f) := j\cdot 2\cdot\pi\cdot f\cdot L$ $\quad XL(fr) = 6.283i \times 10^3$ Reactances

$$XC(f) := \frac{1}{j\cdot 2\cdot\pi\cdot f\cdot C\cdot 10^{-6}}$$

$XC(fr) = -6.283i \times 10^3$

$Z(f) := R + XL(f) + XC(f)$ $\quad Z(fr) = 50$ At resonance

$$rho(f) := \left|\frac{Z(f) - Zo}{Z(f) + Zo}\right|$$

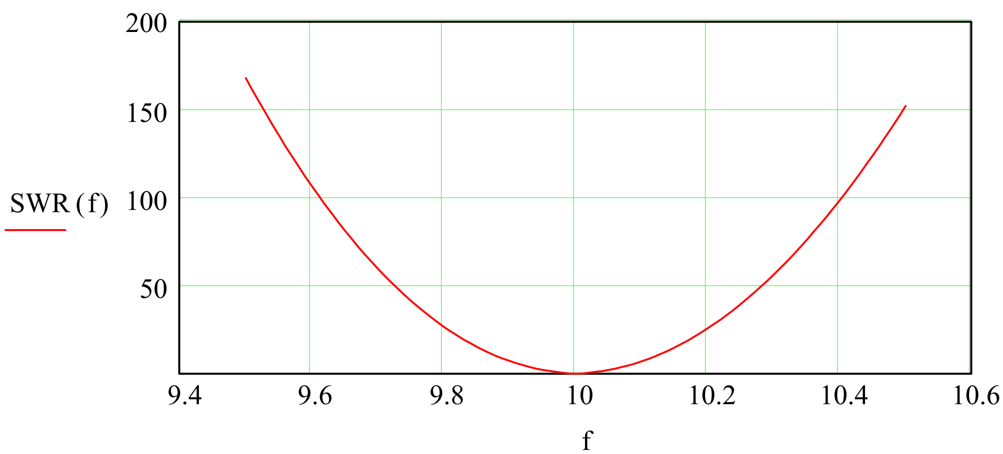
$rho(fr) = 0$ **rho = 0**

$$SWR(f) := \frac{1 + rho(f)}{1 - rho(f)}$$

$SWR(fr) = 1$

The minimum SWR should be 1:1 at fr

$f := 9.5, 9.51 .. 10.5$



Finding the two frequencies F1 and F2 that give SWR = 2.618

$SWRt = 2.618$ target SWR

$Fa := 0.9998 \cdot fr$ start searching for f_low

Given

$SWR(Fa) = SWRt$

$F1(SWRt) := Find(Fa)$ $f\_low := F1(SWRt)$ $f\_low = 9.96029$

$fr - f\_low = 0.0397$

$Fb := 1.0002 \cdot fr$ start searching for f_hi

Given

$SWR(Fb) = SWRt$

$F2(SWRt) := Find(Fb)$ $f\_hi := F2(SWRt)$ $f\_hi = 10.03987$

$f\_hi - fr = 0.0399$

Calculate the Relative Bandwidth Rel_BW (with respect to the resonant freq) at the target SWR

$$Rel\_BW(SWRt) := \frac{F2(SWRt) - F1(SWRt)}{fr} \qquad Rel\_BW(SWRt) = 7.9578 \times 10^{-3}$$

We can calculate the Q at the target SWR = 2.618 (unloaded !)

$$Q := \frac{1}{Rel\_BW(SWRt)} \qquad Q = 125.664$$

Q from the circuit values, we get the same Q value: (unloaded !)

$$Q := \frac{2 \cdot \pi \cdot fr \cdot L}{R} \qquad Q = 125.664$$

$SWRref := 2.618$

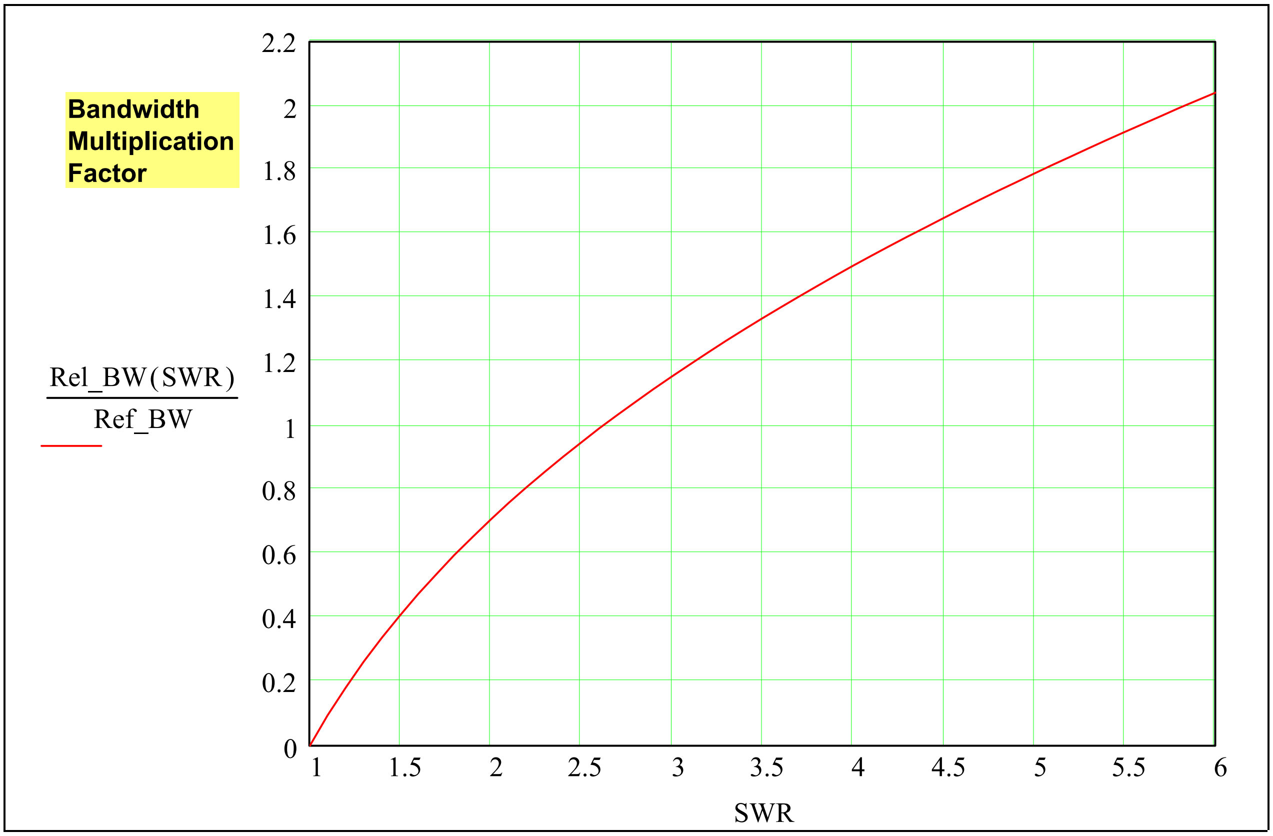
The reference BW is the BW at SWR = 2.618

$Ref\_BW := Rel\_BW(SWRref)$ $Ref\_BW = 7.9576 \times 10^{-3}$

$$\frac{1}{Ref\_BW} = Q$$

Relative BW / Reference BW vs SWR

$\text{SWR} := 1, 1.1 .. 6$



For SWR = 2, the BW is 0.707 times the BW at the reference SWR of 2.618

For SWR = 5.83, the BW is 2.0 times the BW at the reference SWR of 2.618

$\frac{\text{Rel\_BW}(2)}{\text{Ref\_BW}} = 0.7071$ $\frac{\text{Rel\_BW}(5.83)}{\text{Ref\_BW}} = 2$ $\frac{\text{Rel\_BW}(4)}{\text{Ref\_BW}} = 1.5$

Pick the SWR value that you like !

For the **series R L C** circuit, a a random length of low loss 50 ohm coax cable may also be connected between the RLC and the SWR measuring instrument.
With a coax line, the impedance will differ from 50 +/- j 50 ohms at the SWR = 2.618 points, leaving the SWR bandwidth unchanged.

2- Application to a Parallel RLC circuit

The equation describing the parallel RLC circuit is similar to Eq. 1 above, Admittances values may be used directly in Eq. 1. Alternately, the impedance values may be used by taking the inverse of Eq. 1.

This Q measurement technique also works with a parallel RLC circuit, as long as the SWR at resonance is 1:1.
At the SWR = 2.618 points, the normalized impedance is: 0.5 +/- j 0.5 corresponding to: 25 +/- j 25 ohms.
Notice that the coupling of the SWR measuring instrument is 100% in this case.

This technique also works at reduced coupling with a link coupled RLC circuit. The coupling will have to be adjusted for 1:1 SWR at resonance. Also the link may be connected to a random length of low loss, 50 ohm coax cable.
With a link or coax, the impedance at the link or coax end will differ from 25 +/- j 25 ohms, but the SWR will be unchanged.
Using a link with variable coupling allows measuring the parallel R L C circuit Q factor with an SWR analyzer, provided that the link coupling is adjusted for 1:1 SWR at the desired frequency. This method also works with link coupled resonators such as cavities used in duplexers.

3- Using the Series or Parallel RLC circuit as a bandpass filter

Both the the series and parallel circuits will exhibit a lower Q factor when driven by a non zero impedance source, typically a **50 ohm voltage source**.
With Rsource = 50 ohms and R = 50 ohms in the RLC circuit:
- The Q factor degrades by 50 % and the bandwidth doubles.
- The loaded Q = unloaded Q / 2

The bandwidth increase comes from the increased R (in the series circuit for instance) going from 50 to 100 ohms.

Application to dipole antennas exhibiting series resonance

The series R L C model may be applied to dipole antennas which present approx. 50 ohms at the feedpoint. Calculating the antenna Q from the SWR=2.62 is valid even if the dipole is fed by a low loss line.
When the antenna is fed by a 50 ohm source, then its effective bandwidth will double.
Here we still have the loaded Q = unloaded Q / 2
The Q measured from the bandwidth at SWR points of 2.618 gives the unloaded Q.
Measuring at the SWR points of 5.83 will give the loaded Q of the antenna, since the measured SWR bandwidth will be twice the value obtained at the SWR points of 2.618.
This assumes that the TX output impedance and the feedline is 50 ohms.
In a general case, the reflected impedance at the antenna will modify the effective bandwidth and the actual resonant frequency, Note that the SWR meter connected at the TX will NOT show this effect.
Also the bandwidth of the matching circuits at the TX will also affect the effective impedance seen by the antenna and modify its loaded Q and bandwidth.
One way to measure the effective bandwidth of the dipole might be to insert an RF ammeter in series with one dipole leg. Find the frequencies where the current is reduced by ~ 30% (1/sqrt 2) and compute the effective bandwidth this way.

**Reference : Electronic Engineering by Alley / Atwood, John Wiley and Sons Inc.
Chapter 9**

CURVE FIT FOR THE K FACTOR VS SWR

$$K(SWR) := \frac{Rel\_BW(SWR)}{Ref\_BW}$$

SWR =	K(SWR) =
1	$-2.211\cdot 10^{-8}$
1.1	0.095
1.2	0.183
1.3	0.263
1.4	0.338
1.5	0.408
1.6	0.474
1.7	0.537
1.8	0.596
1.9	0.653
2	0.707
2.1	0.759
2.2	0.809
2.3	0.857
2.4	0.904
2.5	0.949

$i := 0, 1 .. 49$

$swr_i := 1.1 + i\cdot 0.1$ SWR vector starts at 1.1 up to 6

$$k_i := \frac{Rel\_BW(swr_i)}{Ref\_BW}$$

APPROXIMATION PAR POLYNOMES D'ORDRE 2 A 9

vecteur y — vecteur x (variable indépendante)

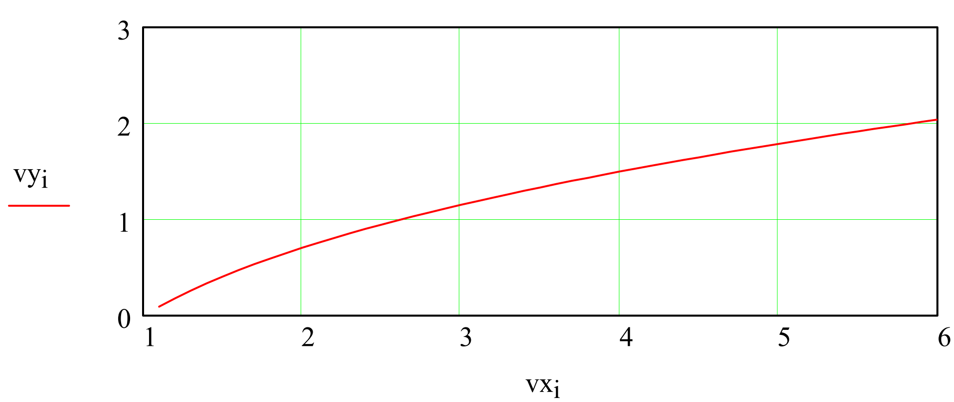
$vy := k$ $vx := swr$

$N := last(vx)$ $N = 49$

$i := 0 .. N$

Pour tracer F(x):

$$nextval := \frac{vx_N - vx_0}{500} + vx_0$$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}$$

Polynôme du deuxième ordre

$A := \mathrm{linfit}(vx, vy, F)$

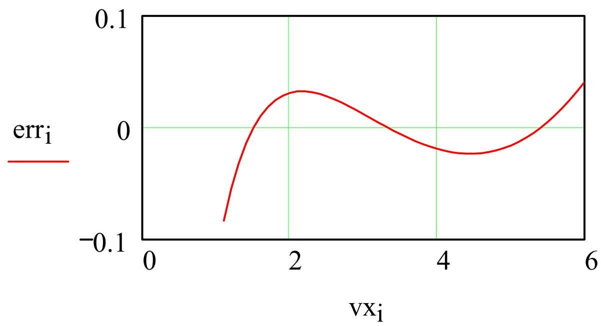
$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2$

$x := vx$

$err := vy - \overrightarrow{F(x)}$

$j := 2$

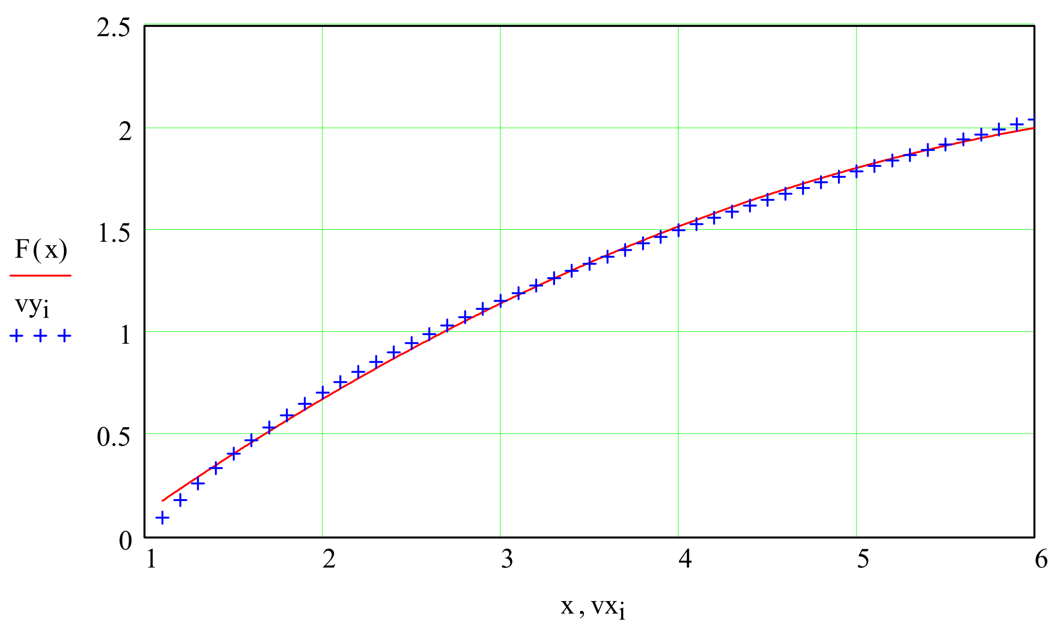
$$errtot_j := \sum_{i=0}^{N} \left(err_i\right)^2 \qquad errtot_2 = 0.031$$



$$A = \begin{pmatrix} -0.529 \\ 0.693 \\ -0.045 \end{pmatrix}$$

Here are the coefficients: Ao, A1, A2

$x := vx_0, nextval .. vx_N$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \end{pmatrix}$$

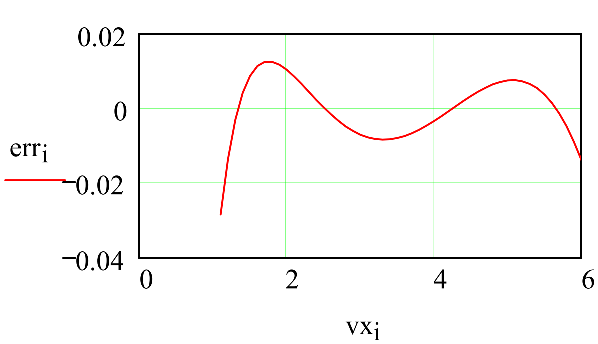
$$A := \mathrm{linfit}(vx, vy, F)$$

$$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3$$

$j := 3 \qquad x := vx$

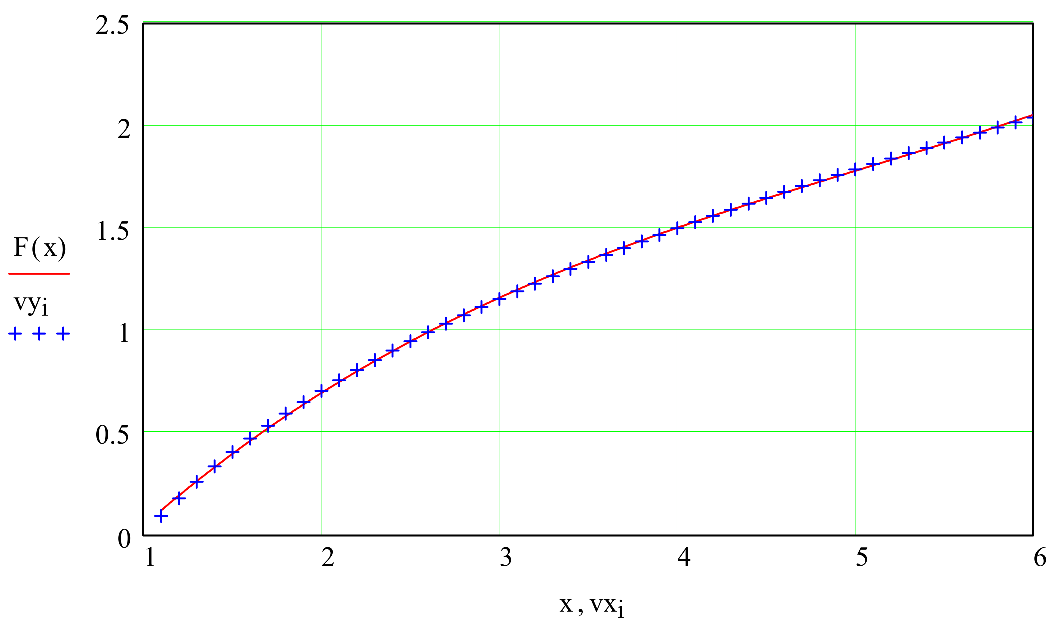
$$err := \overrightarrow{vy - F(x)}$$

$$errtot_j := \sum_{i=0}^{N} (err_i)^2 \qquad errtot_3 = 3.25 \times 10^{-3}$$



$$A = \begin{pmatrix} -0.841 \\ 1.031 \\ -0.151 \\ 9.924 \times 10^{-3} \end{pmatrix}$$

$$x := vx_0, nextval .. vx_N$$



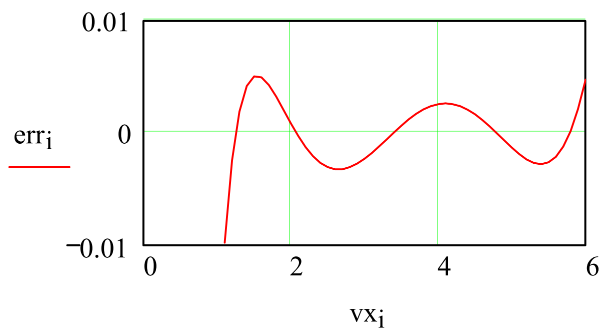
$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \end{pmatrix} \qquad A := \text{linfit}(vx, vy, F)$$

$$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4$$

$$j := 4 \qquad x := vx$$

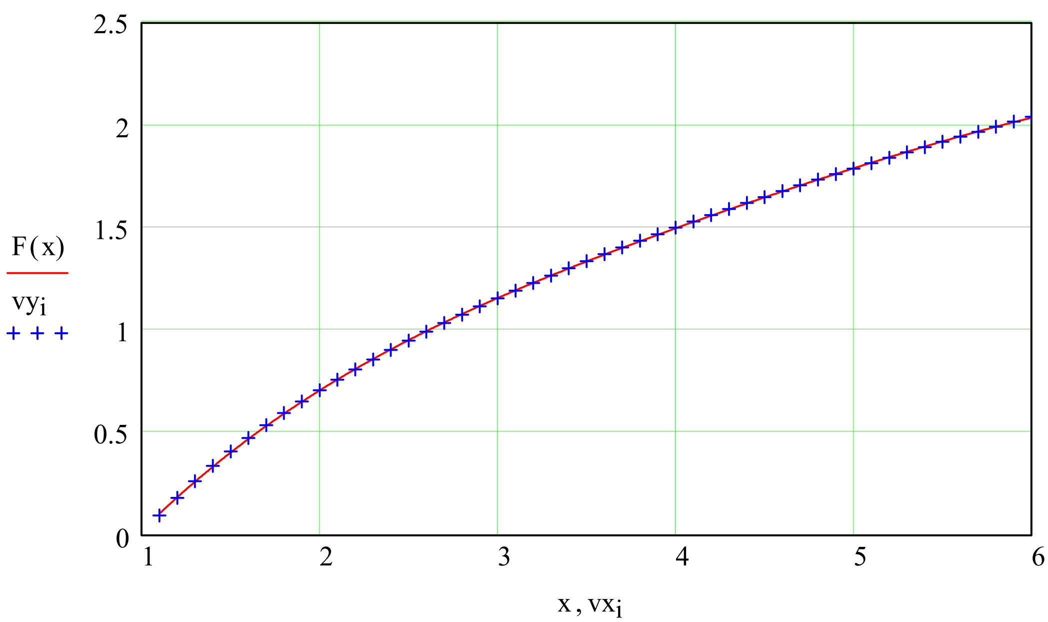
$$err := \overrightarrow{vy - F(x)}$$

$$errtot_j := \sum_{i=0}^{N} \left(err_i\right)^2 \qquad errtot_4 = 3.926 \times 10^{-4}$$



$$A = \begin{pmatrix} -1.083 \\ 1.391 \\ -0.33 \\ 0.046 \\ -2.555 \times 10^{-3} \end{pmatrix}$$

$$x := vx_0, nextval .. vx_N$$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \end{pmatrix}$$

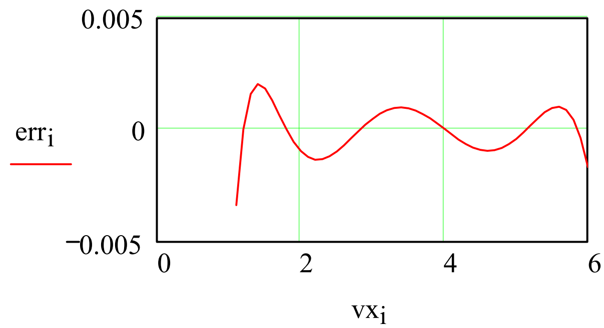
$A := \mathrm{linfit}(vx, vy, F)$

$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4 + A_5 \cdot x^5$

$j := 5 \qquad x := vx$

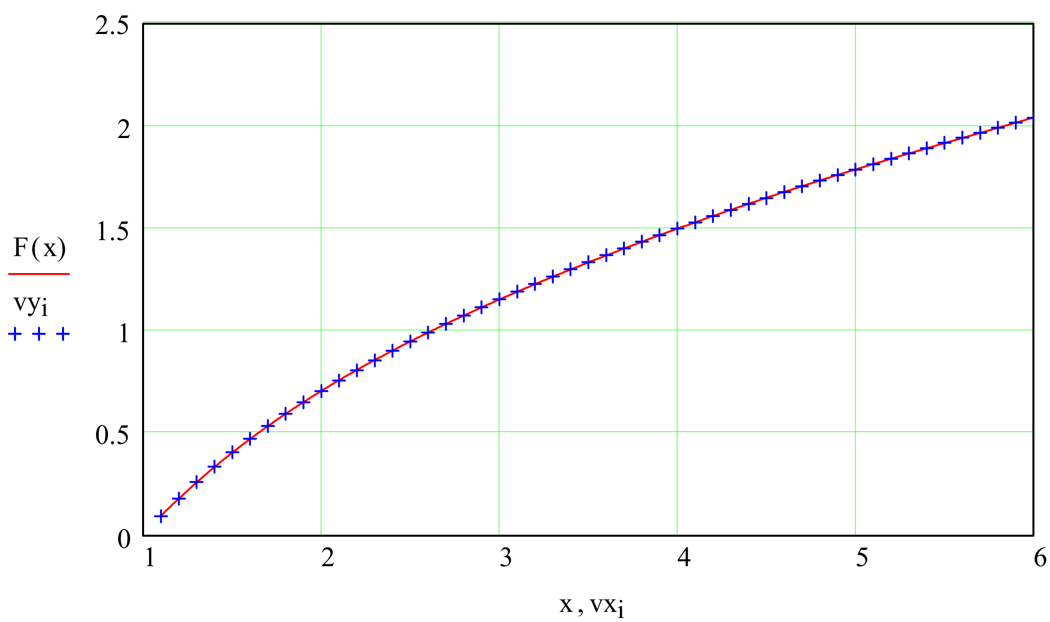
$err := \overrightarrow{vy - F(x)}$

$$errtot_j := \sum_{i=0}^{N} \left(err_i\right)^2 \qquad errtot_5 = 5.077 \times 10^{-5}$$



$$A = \begin{pmatrix} -1.28601 \\ 1.77423 \\ -0.59467 \\ 0.13042 \\ -0.01511 \\ 7.07052 \times 10^{-4} \end{pmatrix}$$

$x := vx_0, nextval .. vx_N$



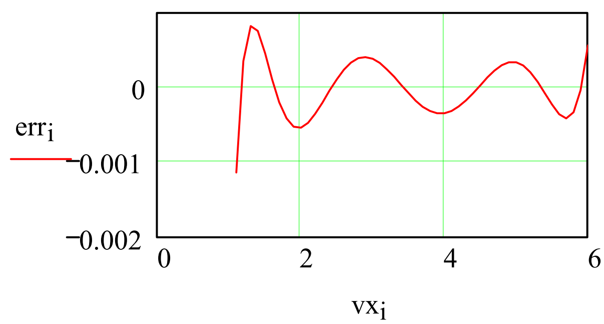
$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \end{pmatrix} \qquad A := \mathrm{linfit}(vx, vy, F)$$

$$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4 + A_5 \cdot x^5 + A_6 \cdot x^6$$

$j := 6 \qquad x := vx$

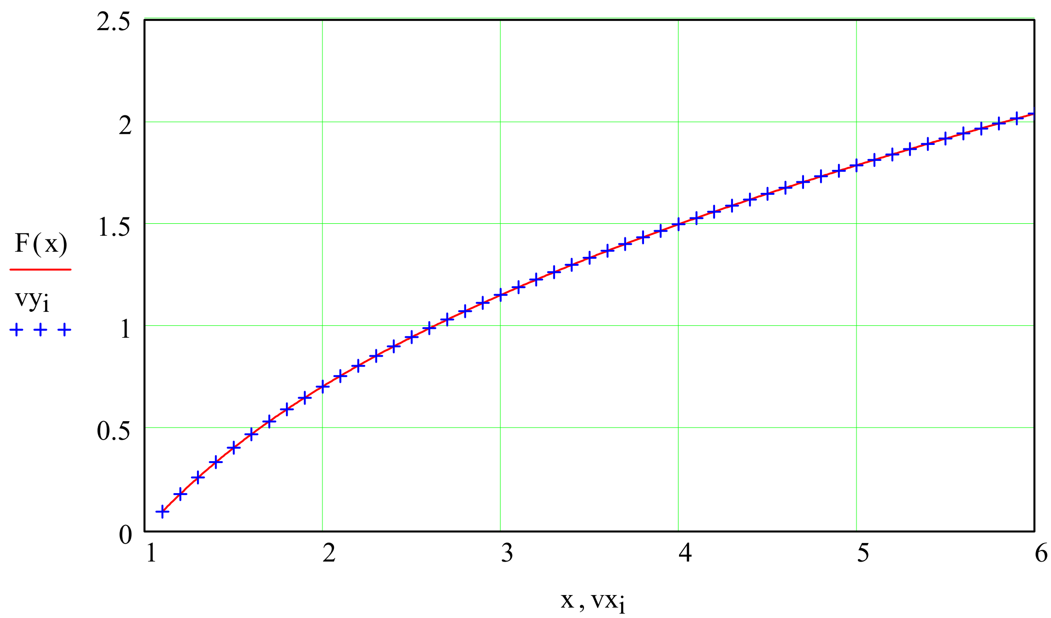
$$err := vy - \overrightarrow{F(x)}$$

$$errtot_j := \sum_{i=0}^{N} (err_i)^2 \qquad errtot_6 = 6.83 \times 10^{-6}$$



$$A = \begin{pmatrix} -1.463 \\ 2.179 \\ -0.953 \\ 0.288 \\ -0.052 \\ 5.043 \times 10^{-3} \\ -2.036 \times 10^{-4} \end{pmatrix}$$

$x := vx_0, nextval .. vx_N$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \end{pmatrix}$$

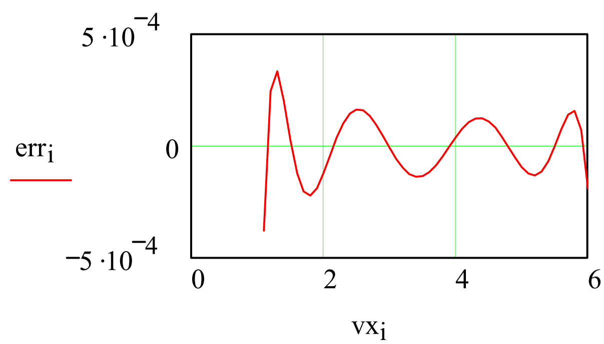
$A := \mathrm{linfit}(vx, vy, F)$

$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4 + A_5 \cdot x^5 + A_6 \cdot x^6 + A_7 \cdot x^7$

$j := 7$ $\qquad x := vx$

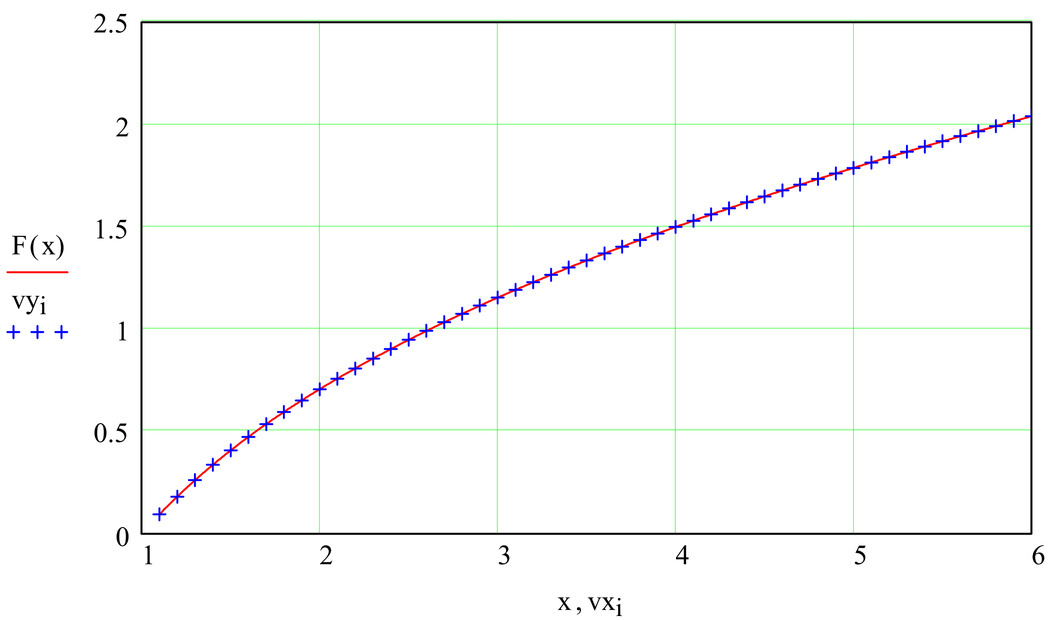
$err := vy - \overrightarrow{F(x)}$

$errtot_j := \sum_{i=0}^{N} (err_i)^2 \qquad errtot_7 = 9.402 \times 10^{-7}$



$$A = \begin{pmatrix} -1.622 \\ 2.605 \\ -1.412 \\ 0.548 \\ -0.135 \\ 0.02 \\ -1.696 \times 10^{-3} \\ 6.006 \times 10^{-5} \end{pmatrix}$$

$x := vx_0, nextval .. vx_N$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \\ x^8 \end{pmatrix}$$

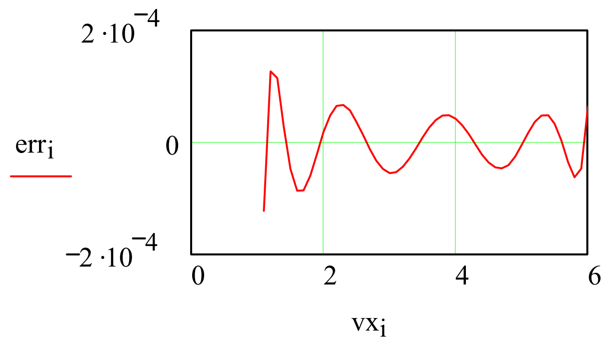
$$A := \mathrm{linfit}(vx, vy, F)$$

$$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4 + A_5 \cdot x^5 + A_6 \cdot x^6 + A_7 \cdot x^7 + A_8 \cdot x^8$$

$j := 8 \qquad x := vx$

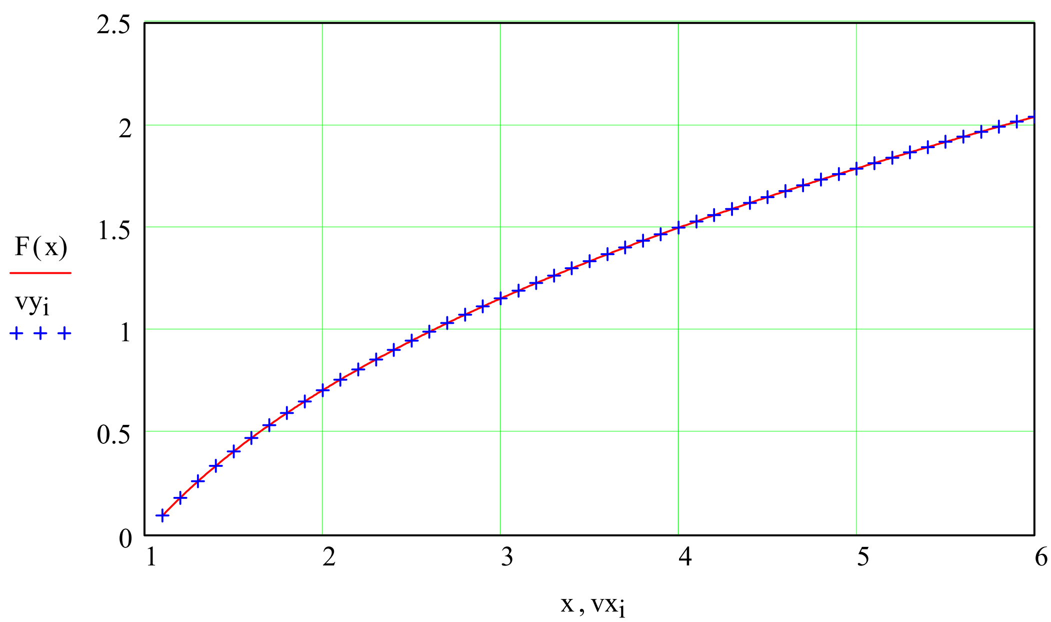
$$err := vy - \overrightarrow{F(x)}$$

$$errtot_j := \sum_{i=0}^{N} \left(err_i\right)^2 \quad errtot_8 = 1.315 \times 10^{-7}$$



$$A = \begin{pmatrix} -1.767 \\ 3.051 \\ -1.98 \\ 0.94 \\ -0.297 \\ 0.061 \\ -7.84 \times 10^{-3} \\ 5.713 \times 10^{-4} \\ -1.8 \times 10^{-5} \end{pmatrix}$$

$$x := vx_0, nextval .. vx_N$$



$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \\ x^8 \\ x^9 \end{pmatrix}$$

$A := \mathrm{linfit}(vx, vy, F)$

$F(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + A_4 \cdot x^4 + A_5 \cdot x^5 + A_6 \cdot x^6 + A_7 \cdot x^7 + A_8 \cdot x^8 + A_9 \cdot x^9$

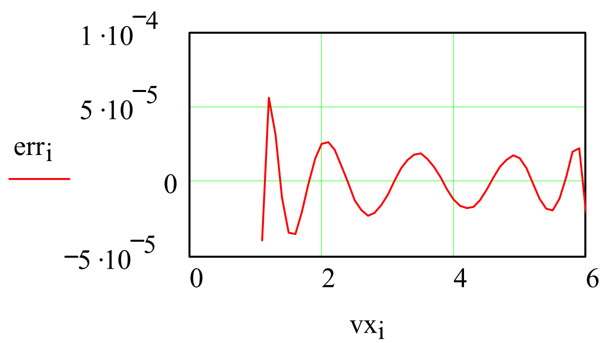
$j := 9$ $\qquad x := vx$

$err := vy - \overrightarrow{F(x)}$

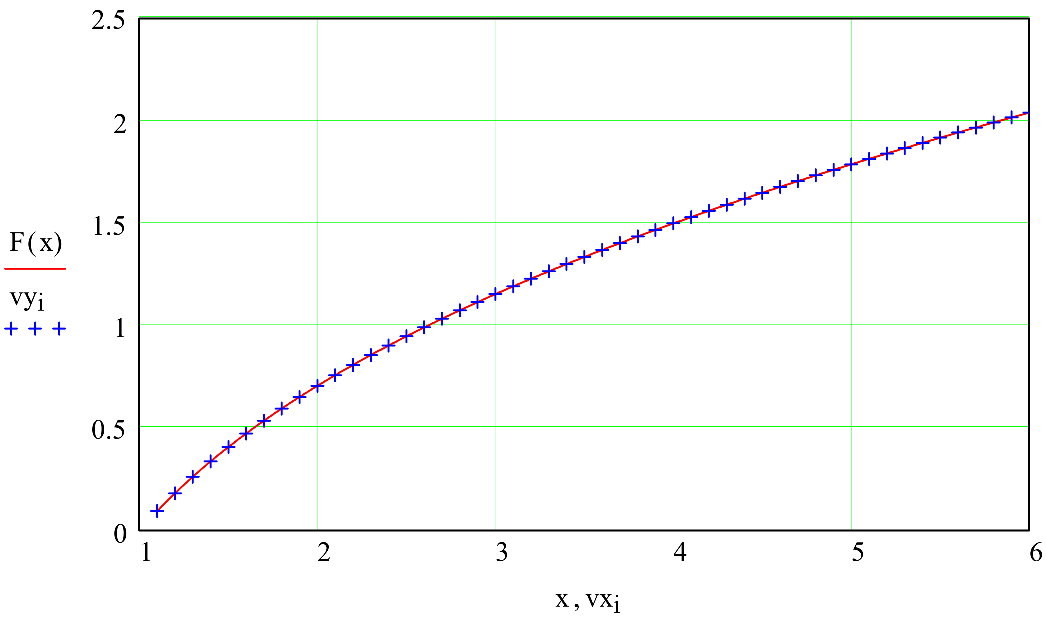
$errtot_j := \sum_{i=0}^{N} \left(err_i\right)^2 \qquad errtot_9 = 1.836 \times 10^{-8}$

$A =$

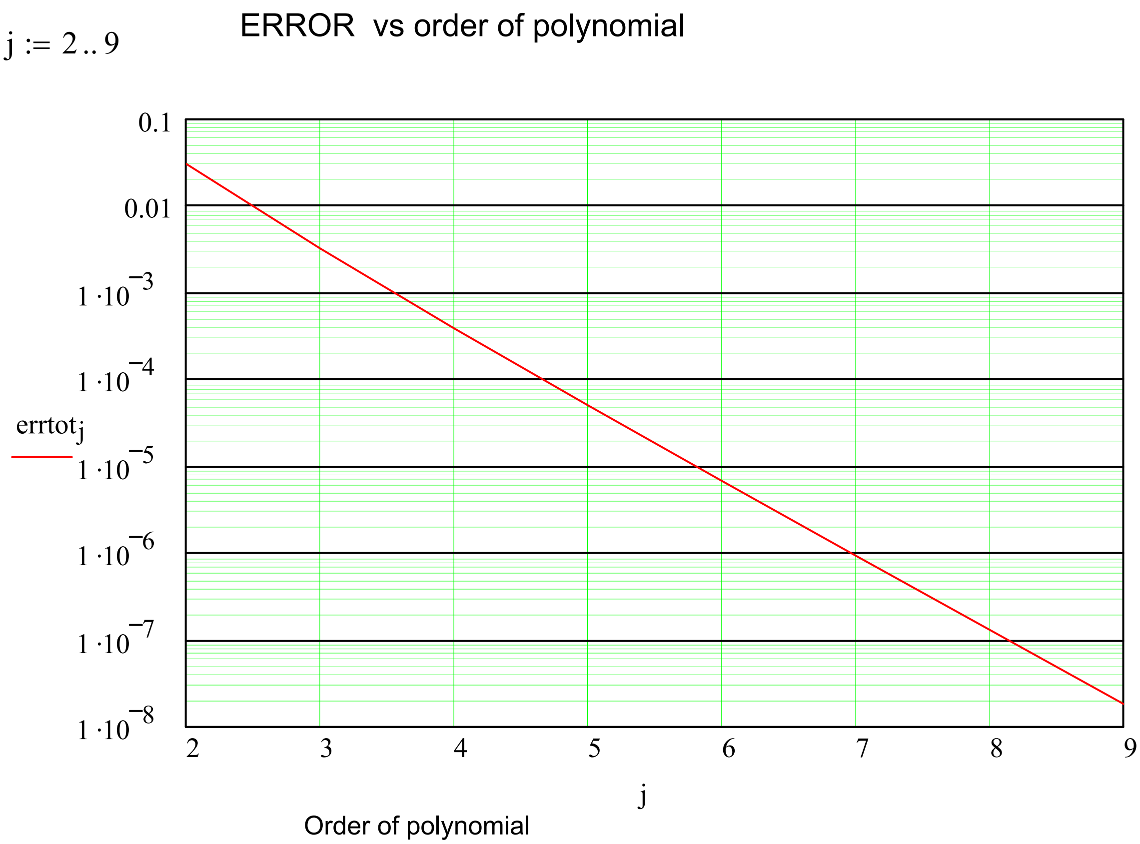
	0
0	-1.901
1	3.516
2	-2.664
3	1.5
4	-0.578
5	0.152
6	-0.027
7	$2.981 \cdot 10^{-3}$
8	$-1.927 \cdot 10^{-4}$
9	$5.469 \cdot 10^{-6}$

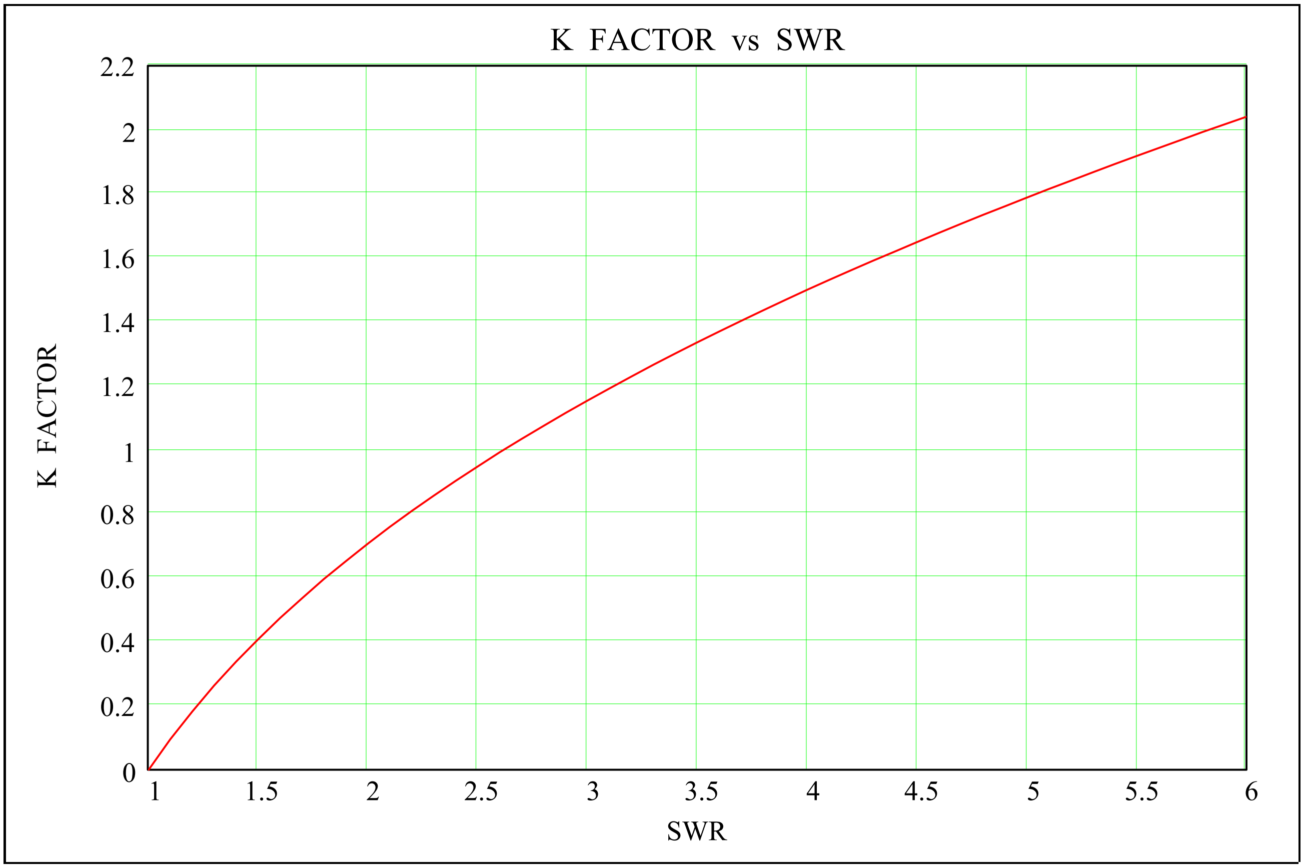


$x := vx_0, nextval \,..\, vx_N$



$j := 2 .. 9$


ERROR vs order of polynomial
0.1
0.01
1 ·10−3
1 ·10−4
1 ·10−5
1 ·10−6
1 ·10−7
1 ·10−8
errtotj
2
3
4
5
6
7
8
9
j
Order of polynomial


K FACTOR vs SWR
K FACTOR
SWR
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6

Examples of Q Measurement

$SWRm := 4$ $K := 1.5$ From graph, K(SWRm) = 1.5 at Measured SWRm

$BW1 := (F2(SWRm) - F1(SWRm))$ Bandwidth between the two SWR points: SWRm

$Q := \frac{K \cdot fr}{BW1}$ $Q = 125.664$

$SWRm := 2$ $K := 0.7071$ From graph, K(SWRm) = 0.7071 at Measured SWRm

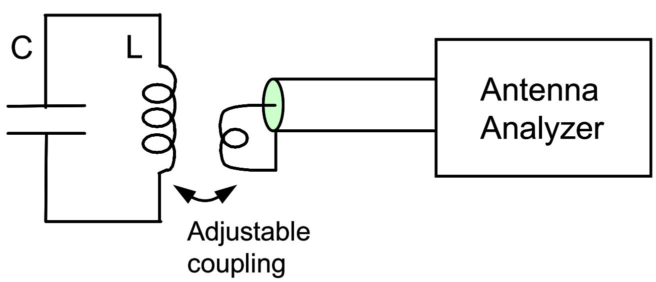
$BW1 := (F2(SWRm) - F1(SWRm))$ Bandwidth between the two SWR points: SWRm

$Q := \frac{K \cdot fr}{BW1}$ $Q = 125.662$

IF BW1 is measured from resonance (SWR=1:1) to the measured SWR point: SWRm

$K(SWRm) = \frac{Rel\_BW(SWR)}{Ref\_BW}$ K(SWRm) is obtained from graph above

$$\boxed{Q = \frac{K(SWRm) \cdot fr}{BW1}}$$



The antenna analyzer / SWR meter is connected to a short 50 ohm coax cable with a link coil at the end.
Adjust the coupling, and frequency, L or C to obtain 1:1 SWR at the desired resonant frequency (**fr**) as set by L and C.
Change the frequency (up or down) to get an SWR between 2 and 6. Note as **SWRm**.
Note the frequency as **f.**
Compute **BW1 = f - fr** Take the absolute value.
From the graph find the **K** factor corresponding to **SWRm**.

Compute the **Q** of the LC circuit:

$$Q = \frac{K \cdot Fr}{BW1} \qquad Q = \frac{K \cdot fr}{|f - fr|}$$

Calculate LC circuit Parameters

Assume C is known

$$L = \frac{10^6}{4\cdot\pi^2\cdot f^2\cdot C}$$

f in MHz
C in pF
L in uH

$L = 100$

$f := 10$

$C = 2.533$

$$ESR := \frac{2\cdot\pi\cdot f\cdot L}{Q}$$

$ESR = 50$

$Rpar := Q\cdot 2\cdot\pi\cdot f\cdot L$

$Rpar = 7.896 \times 10^5$

$f := f\_low$ $f := f\_hi$ $fr := 10$ $SWR := SWRt$

$f = 10.039868$ $SWR = 2.618$

Since fr = sqrt (f * fx)

$$fx := \frac{fr^2}{f}$$

At **f** and **fx** the SWR has the same value.

$|f - fx| = 0.08$

f - fx = BW, the bandwidth at equal SWR points

$$Q = \frac{K \cdot fr}{BW1}$$

$$Q := \frac{fr \cdot \left(1.77423 \cdot SWR - 0.59467 \cdot SWR^2 + 0.13042 \cdot SWR^3 - 0.01511 \cdot SWR^4 + 0.000707 \cdot SWR^5 - 1.28601\right)}{\left| f - \frac{fr^2}{f} \right|}$$

$Q = 125.721$

NOTE : At Q=300 a 1% change in SWR gives a 1% change in Q
A 1% increase in the freq. difference gives ~ -1% change in Q